Gadanidis, G., Cendros, R., Floyd, L., & Namukasa, I. (2017). Computational thinking in mathematics teacher education. *Contemporary Issues in Technology and Teacher Education, 17*(4), 458-477.

Computational Thinking in Mathematics Teacher Education

George Gadanidis
Western University

Rosa Cendros
Western University

Lisa Floyd
Western University

Immaculate Namukasa
Western University

As computational thinking (CT) is increasing in focus in K-12 education, it is important to consider how teacher education programs may better prepare teacher candidates (TCs). Previous studies have found that TCs do not always have a firm understanding of what CT involves, and they might not have clear ideas about how to develop CT in their future classrooms. In this context, the authors developed a course for elementary school TCs focusing on CT in mathematics education. The course integrated CT in the context of mathematics activities to help TCs develop both a conceptual understanding of mathematics and mathematics teaching with CT. The paper presents a case study analysis of TCs' online discussions and reflection assignments of the course, as well as themes in their learning about and attitudes toward CT in mathematics teaching and learning.

Computational thinking (CT) can take various forms: It can be screen-based computer programming, it can be used to control digital circuits and robots, and it can more generally be the design of algorithmic solutions to problems that can be carried out by a computer. K-12 education has become a focal point for teaching CT. However, it is not clear how CT may or should fit within K-12 education (other than traditional computer science courses, typically offered to high school students).

How do mathematics educators make room in an already crowded curriculum? Do they teach CT as its own subject or integrated with other subjects? Should they follow Papert's approach by integrating CT within mathematics education? How do they address CT in teacher education? This paper relates to the last two questions by investigating 143 elementary teacher candidates' (TCs') experiences in an 18-hour course on CT in mathematics education.

Wing (2006) suggested, “To reading, writing, and arithmetic, we should add CT to every child’s analytical ability” (p. 33). Barr and Stephenson (2011) stated that because today’s students are living – and will continue to live – in a world heavily influenced by computing, “It is no longer sufficient to wait until students are in college to introduce these concepts” (p. 49). As Bower and Falkner (2015) wrote, “We need to ensure that our educational systems provide not only the fundamentals of digital literacy – familiarity with the tools and approaches to interacting with technology – but also the CT processes needed to understand the scientific practices that underpin technology” (p. 37).

As CT is increasing in focus in K-12 education, it is important to consider how teacher education programs may better prepare TCs. According to Barr and Stephenson (2011), teacher professional development and the education of TCs are critical elements to successful implementation of CT in K-12 education. However, Bower and Falkner (2015) found that TCs do not always have a firm understanding of what CT involves, and they might not have clear ideas about how to develop CT in their future classrooms. Also, in Bower and Falkner’s (2015) study, TCs expressed their need to develop “computational thinking pedagogical capabilities – understanding of the curriculum, lesson ideas, strategies for implementation, links to real world examples” (p. 44).

Gadanidis (2017) suggested that CT offers five affordances that support elementary mathematics education: agency, access, abstraction, automation, and audience. Furthermore, according to Yadav, Mayfield, Zhou, Hambrusch and Korb (2014), “It is important that we develop teachers’ understanding of computational thinking in the context of the subject matter they teach”; otherwise, “teachers may only gain an ‘abstract’ understanding of CT,” and “their knowledge will remain inert and they will be unable to incorporate it into their teaching” (p. 14).

In this context, we developed a course focusing on CT in mathematics education for elementary school TCs, which we refer to as the Course. The Course integrates CT in the context of mathematics activities to help TCs develop conceptual understanding of mathematics and mathematics teaching with CT. Consistent with Yadav et al. (2014) and Bower and Falkner’s (2015) guidelines, the Course provides connections to the real world, as well as lesson ideas and pedagogic examples to integrate CT in the context of mathematics as a subject matter.

In this paper, we present our analysis of TCs' online discussions and reflection assignments while participating in the Course. Through this analysis, we addressed the following questions:

- What do TCs learn about CT in mathematics education?
- What attitudes do TCs develop towards CT in mathematics teaching?
- What role do the online resources and experiences play in the first two questions?
- What role do the face-to-face experiences play in the first two questions?

Context

Elementary school (K-6) TCs in our teacher education program typically do not have a strong mathematics or computer science background, and in fact, many of them hold negative views toward mathematics. Most of our TCs have education backgrounds from the humanities, social studies, and the arts and not from the sciences. In this context, it is interesting to investigate what they learned and what attitudes they developed in a course that integrates mathematics and CT.

Participants were 143 elementary school TCs who agreed to participate in the research, out of a total of 157, distributed among five sections of the Course. The Course had a duration of 9 weeks, 2 hours per week, where the five odd numbered sessions were face to face, and the four even numbered sessions were online. Following is the weekly outline of topics:

- Week 1 (on site) and Week 2 (online) – Algorithms, coding, and CT in the context of geometry
- Week 3 (on site) – Abstraction and CT in the context of coordinate geometry
- Week 4 (online) and Week 5 (on site) – CT in the context of probability
- Week 6 (online) and Week 7 (on site) – CT in the context of patterning and algebra
- Week 8 (online) and Week 9 (on site) – CT and mathematics pedagogy in the context of measurement and number sense.

The face-to-face sessions consisted of hands-on learning, using different coding platforms and digital tangibles to develop coding activities that support mathematics teaching and learning. The online component consisted of two elements: (a) readings, viewings and activities related to each week's topic, and (b) collaborative knowledge construction and reflection in small groups (four to seven participants) of mind-maps through the online tool Popplet (popplet.com), which replaced the more-traditional text-based discussion forum. Figure 1 shows an example of the mind-maps TCs created through Popplet.

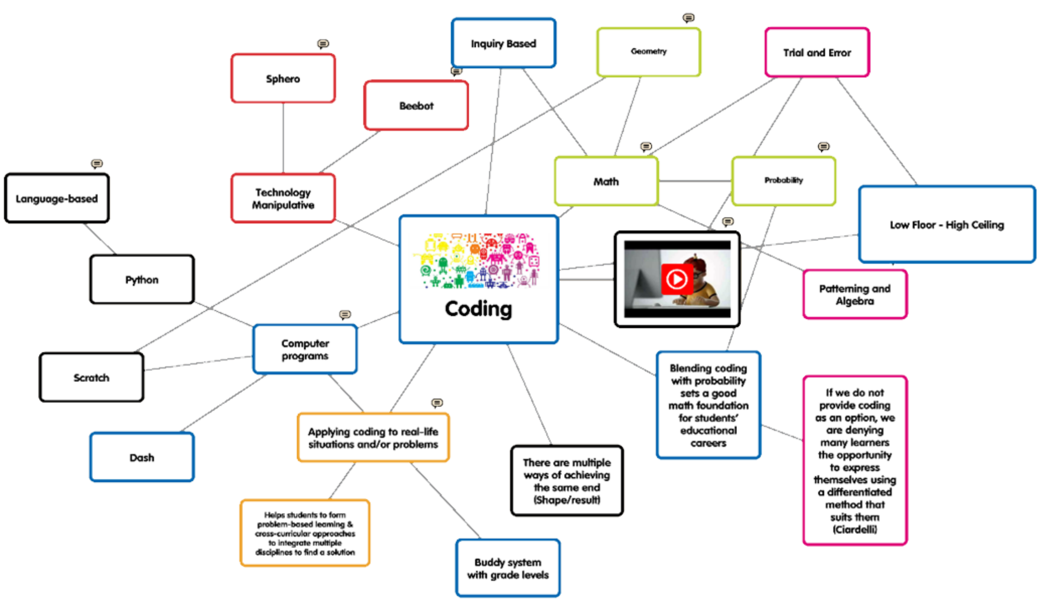

Figure 1. *An example of the mind-maps TCs created through Popplet.*[/caption]

There was a total of 31 small groups across five sections. The total number of mind-maps analyzed for this study was 93, distributed as follows:

- Week 2: 31 mind-maps
- Week 4: 31 mind-maps
- Weeks 6 and 8: 31 mind-maps

Prior to each online week, TCs received a link with access to their group's mind-map, which was initially blank. For Weeks 6 and 8 each group used only one canvas, so for Week 8 TCs connected ideas and topics within the mind-map they used on Week 6. The prompts used by the instructor (third author) to guide TCs to develop the mind-maps included an explanation on how Popplet can help TCs make connections between the online and in-class activities, a list of suggested – not mandatory – topics to address in the mind-map, and a video on how to use the tool.

Additionally, the Course included a reflection assignment to be submitted online by Week 5, where TCs were required to write a 500-word reflection on one of three options: (a) discuss teaching and learning connections of a CT/math resource not listed in the Course outline, (b) critically reflect on a theme or activity of the Course based on their experience, or (c) write a short paper on a math and computational thinking topic. The data for this analysis was obtained from the Popplet mind-maps developed during the online weeks, and the 500-word reflections submitted by TCs in Week 5. The total number of assignments analyzed was 143.

Theoretical Framework

We adopt a sociocultural perspective of knowledge, constructed in interactions with others (Vygotsky, 1978). By "others" we also refer to the technology that permeates our culture. Technology is not simply a tool for human intention, but rather an actor in the cognitive ecology of immersive humans-with-technology environments (as described by Levy, 1993, 1998) that serves to not only support but also disrupt and reorganize human thinking (Borba & Villareal, 2005).

Actor-network theory (Latour, 2005) emphasizes the reciprocal relationship among actors, where they are both acting and acted upon (Thumlert, deCastell & Jensen, 2014). By "actors" we refer to the various technologies that surround individuals, including the digital artifacts of the new media world. Just as importantly we also refer to the human knowledge, ideas, beliefs, methods, and specialized processes for perceiving and acting in the world.

In this case study, we strove to identify the actors that TCs came to think within their learning process as they engaged with the various ideas and teaching practices in relation to CT in mathematics education depicted in online and face-to-face learning resources and activities. Although the Course engaged teachers with various CT technologies, in this study we focused on TC thinking about their experiences with CT. To identify actors that played a role in TC thinking we searched for themes that emerged in the mind-maps and reflection assignments that TCs completed as part of their work in the Course.

Foreshadowing the study's findings, we identified several actors that TCs came to think with in the Course: (a) CT-actors (e.g., CT-math integration, Scratch, the visual coding platform developed by the Massachusetts Institute of Technology [available at scratch.mit.edu] in math teaching, CT in other subjects, or CT in society); b) attitude-actors (e.g., concern and anxiety in using CT in math and teaching or a growth mindset);

(c) online-resource-actors (e.g., facilitating TC learning or modelling TC teaching); and (d) face-to-face-experience-actors (e.g., positive attitude or conceptual understanding).

Method

Our study employed a case study approach, which is suitable for collecting in-depth stories of teaching and learning and studying a bounded system (that is, the thoughts and actions of participants in a particular education setting) so as to understand it as it functions under natural conditions (Stake, 2000a, 2000b; Yin, 1994). The five Course sections were treated as a single case, as TCs participated in the same online environment with the same tasks to complete.

Guided by the questions of our study, we used qualitative content analysis (Berg, 2004) to identify what TCs learned, their attitudes toward the topics, and comments about their experience with the on-site and online activities. First, we extracted all text data from the mind-maps and reflection assignments for participating TCs. Then, we used a manual content analysis by reading all the discussions and assignments, while coding the text into four categories based on the questions of the study: (a) TC learning and (b) attitudes about CT in mathematics education, and the role of (c) online resources and (d) face-to-face activities. Not every mind-map group and not every student assignment commented on all four categories of analysis (see Table 1).

Table 1
Mind-Map and Reflection Assignment Frequency in Each Category

Category	Week 2	Week 4	Weeks 6 and 8	Reflection Assignment	Total
TC learning	31	31	31	143	236
TC attitudes	30	24	10	110	174
Online resources	28	20	15	87	150
Face-to-face activities	10	8	11	66	95

Finally, within each of the four categories, we identified emergent themes. The themes were obtained by contrasting our manual analysis with the topics extracted by the qualitative analysis software Wordstat. In the process, some themes extracted by the software were integrated to match the themes we identified manually (for example, the themes “cross-curricular” and “mathematics” were integrated into the theme “CT and mathematics education”). The software also allowed us to obtain the frequency of appearance of each topic.

However, to count frequencies we chose to use the indicator “case appearance” instead of the “absolute frequency” of a theme, as we realized that once a topic was initially introduced in a mind-map, several TCs in the group would comment on it. For this reason, the appearance of a theme counted only once per mind-map. In the same way, the appearance of a theme counted only once per reflection assignment, as this was an individual activity and it resulted in a better indicator of how many TCs considered that a topic was relevant and worth mentioning. In other words, each mind-map and each reflection assignment was considered a case, and we counted how many cases contained a particular theme. The total number of cases was 236 (see Table 1).

To strive for trustworthiness in our findings, we used four types of triangulation. In the first place, we used triangulation by data source (as recommended by Miles, Huberman, & Saldaña, 2014), which led us to include the reflection assignments as an additional source of data to confirm the information obtained from the mind-maps and provide more insight into individual TCs' learning process.

Second, we used investigator triangulation (Denzin, 1989). We included two researchers external to the case study who collected and analyzed the data (the first two authors), a researcher involved in the case study in the role of instructor who provided more insight and assessed the outcomes of the study as compared to her experience (the third author), and a researcher who focused on the validity of the data, methods and findings, absent of bias towards the topic (the fourth author). Among all researchers, the categories of analysis and emergent themes were discussed, confronted, and challenged to achieve consensus.

Third, we included a triangulation of data type (Miles et al., 2014) or multimethod triangulation (Meijer, Verloop & Beijaard, 2002), by using both frequencies analysis and qualitative data analysis to obtain our results. Through this process, we complemented the quantitative findings with a thick, rich description of the phenomena (Cresswell & Miller, 2000).

Finally, we included theory triangulation (Denzin, 1989; Miles et al., 2014), by which we discussed our results through the lenses of different theories and previous research. In some cases what TCs say in course settings may be affected by both the instructor's beliefs and the course orientation, thus, introducing a bias in research data and limiting the trustworthiness of a study's findings.

Findings and Discussion

In this section we elaborate on the four categories that reflect our four research questions: CT and mathematics pedagogy; attitudes toward CT, mathematics and teaching; role of online resources; and role of face-to-face experiences. The pervasive themes within each of these categories are what we consider to be the actors that TCs came to think with.

CT and Mathematics Pedagogy

As noted in the context section, each week of the Course addressed different topics and approaches for integrating CT in mathematics education. Table 2 shows the frequencies of appearance of themes throughout the mind-maps and assignments. For this study, we only considered *pervasive* themes. By pervasive themes, we refer to themes that persisted in mind-maps in each of Weeks 2, 4 and 6/8 and in the reflection assignment (see Table 2). The pervasive themes were (a) integration of CT and math education; (b) the Scratch coding environment; (c) CT in other subjects; and (d) CT importance in society. These pervasive themes are the CT-actors that TCs came to think with. These actors are evident in the ideas shared and discussed by TCs. For the vast majority of TCs, these were new actors to think with, as most TCs came to the course with little or no knowledge of CT and of its application in mathematics education.

Table 2
Frequency of Expressions Related to CT and Math Pedagogy

Theme	Week 2	Week 4	Weeks 6 and 8	Reflection Assignment	Total
Math-CT integration	31	24	26	143	224
Scratch in math teaching	17	9	6	72	104
CT in other subjects	22	20	10	79	131
CT in society	25	22	24	129	200

CT-Math Integration. The key idea of the Course was the integration of CT with math learning, instead of having CT stand as a separate and disconnected topic. This key idea was also a main discussion topic in Weeks 2 and 8 and was closely related to the prompt for the reflection assignment, which was explained in the context section. Thus, not surprisingly, the integration of CT and mathematics education was a predominant theme/actor in online discussions and in the reflection assignment, present in all 31 mind-maps in Week 2 and in all 143 reflection assignments (see Table 2).

During the first online week of the Course (Week 2), TCs were presented with Papert's (1980) ideas from his book *Mindstorms*: *Children, Computers, and Powerful Ideas*. He said that "learning to communicate with a computer may change the way other learning takes place," and "it is possible to design computers so that learning to communicate with them can be a natural process" (p. 6). Papert developed Logo as a mathematics learning environment accessible to young children. He wrote that Logo "is to learning mathematics what living in France is to learning French" (p. 6).

During Week 2, eight TCs (in a total of five mind maps) identified the process of communicating with computers through code as a mathematical process, and they related it to mathematics education. For example, one student said, "Students who learn how to code will not only learn how to program computers but also to see math as a powerful language -- and not something that is to be feared." Additionally, 87 (60.83%) TCs commented on Papert's ideas in their reflection assignments.

Through these discussions, TCs thought with CT-math integration and engaged in important considerations of how and why to include CT in the curriculum. As Voogt, Fisser, Good, Mishra and Yadav (2015) stated, "It becomes important to consider where CT belongs in the K-12 curriculum; should it be as a computing subject on its own or should we embed CT across other subject areas?" (p. 722).

Scratch for Math Learning. The Scratch coding environment was introduced in the first week of the Course in connection with investigating the dynamical modelling of math concepts and relationships. Additional CT tools (such as programmable robots and circuits) and other coding languages (such as Python) were then added. Scratch, however, continued to be a popular theme/actor in each week in the mind-maps. Seventy-two TCs (50.3%) chose Scratch, or their experiences around it, for developing their reflection assignment. These TCs identified benefits for students using Scratch in their classroom math learning, and 46 of them related Scratch to the development of a growth mindset and the possibility of math learning through trial and error. For example, one student said, "Programs such as Scratch give students the opportunity to see their thinking and manipulate their thinking, allowing them to navigate problems through trial and error."

This idea was also recognized by Bers, Flannery, Kazakoff and Sullivan (2014) as being significant when children are coding, as it supports the view that "children are not expected to 'get it right' the first time" (p. 149) and helps students to accept that failure will lead to success.

Furthermore, evidence that TCs not only talked about but also engaged with Scratch was found in the mind-maps, where they shared web links to the programs they made using Scratch and received feedback from other TCs. Figure 2 shows an example of a Scratch program shared in a mind-map. A total of 26 Scratch programs were shared this way.

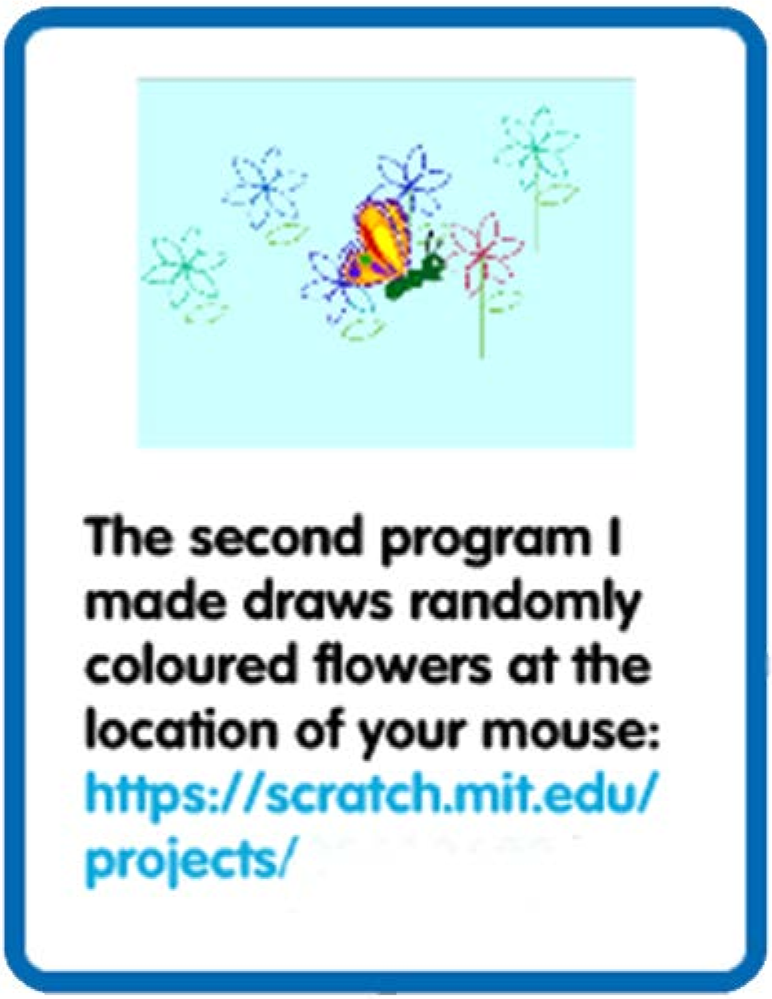

Figure 2. *Example of the resulting shape of a Scratch program shared through a mind-map.*

Although some TCs initially expressed some apprehension in engaging with coding in Scratch and with CT, in general, their comments about Scratch were overwhelmingly positive. This result is consistent with what other studies have found about the implementation of Scratch. For example, Clark, Rogers, Spradling and Pais (2013) found that even teachers who were initially skeptical of implementing computing thought that Scratch was a valuable and easy-to-use tool for their classes. Other research has found that the use of Scratch not only increased computational skills (Baytak & Land, 2011) but also improved learners' attitudes about computing and computer science (Lambert & Guiffre, 2009).

CT in Other Subjects. As shown in Table 2, another common theme/actor in discussions and reflection assignments was seeing CT as also integrated with other subjects. Seventy-nine (55.2%) TCs commented in their reflection assignments about this theme, and the cross-curricular approach was mentioned in 32 mind-maps. Through experiences they shared in class and online, TCs were able to find cross-curricular connections of coding activities and programs, such as Scratch. Following are three representative TC comments on this theme:

> I was also delighted to discover that Scratch is not limited to basic mathematical concepts, but offers the opportunity to explore subjects such as visual arts and music. This not only provides students with new possibilities to explore, but also encourages deeper thinking as to how computational thinking applies to so much more than mathematics.
>
> Coding also encourages an environment that breeds creative and logical students.
>
> Computational thinking can be incorporated into various subjects: language arts, music, art, science, social studies and mathematics. It is important for teachers and teacher candidates to understand computational thinking is not limited to computer scientists and mathematicians.

Consistent with TC comments, Mishra and Yadav (2013) argued that CT can foster creativity because students not only act as consumers of tools but also as builders of technology that is relevant to their lives and society. In Knochel and Patton's (2015) words, "Computational thinking and the writing of code is not just a technical practice implementing mathematical algorithms, but rather a process of design, an act of free speech, and a digital production method" (p. 34).

CT in Society. Related to the previous topic, many TCs made statements in support of the integration of computational thinking in the curriculum (Table 2). These statements were typically based on the need to prepare children for a technological world. Computational thinking and coding were seen by TCs as important skills in today's society that need to be supported by the curriculum. Following are some representative comments for this theme/actor:

> These past couple of weeks have allowed me to realize that thinking computationally is an important quality to maintain, especially in a society that is surrounded with technology.
>
> In society, there is a high demand for workers in computer science. This is why it is important for students to learn about it early on in their education.
>
> Computational thinking is found everywhere in today's digital society, and computing technologies have saturated our lives. In order to prepare today's students, teachers and teacher candidates need to learn how to implement

> computational thinking into core subject areas taught to students, kindergarten to grade 12.

TCs' comments echo Papert's (1980) belief that young students need new cognitive models to be able to respond to the needs of the 21st century. More recently, Bower and Falkner (2015) argued that "preparing students to engage in current technologies and participate as creators of future technologies requires more than is currently being provided" (p. 37).

Attitudes Toward CT, Mathematics, and Teaching

TCs' expressed attitudes in relation to CT evolved throughout the Course. Table 3 shows how the expressions describing TCs' attitudes were distributed among the mind-maps and assignments.

Table 3
Frequencies of Expressions Related to CT

Types of Expressions	Week 2	Week 4	Weeks 6 and 8	Reflection Assignment	Total
Concern and anxiety toward CT and math education	16	10	2	87	115
A growth mindset for teachers and students	30	20	12	105	167

Concern and Anxiety. At the beginning of the Course, TCs expressed concerns about coding and CT, generally, and about their role as teachers integrating CT in their lesson plans and in their teaching. A total of 16 mind-maps during Week 2 included comments about TCs having little to no exposure to coding and expressing feelings of anxiety about the content of the Course. Following are some sample comments about these initial attitudes/actors:

> I honestly thought that this course was going to be about computer programming, and I was dreading it.

> I am very new to coding. In fact, I had never heard of it up until this semester.

> I felt very apprehensive about taking [CT in math education].... I would not consider myself a very strong math student or mathematician, and I have not had an opportunity to learn the concept of coding prior to this course.

This initial state of mind in TCs was expected, as studies have identified increased anxiety and concern in teachers regarding teaching about unfamiliar content (Curzon, McOwan, Cutts, & Bell, 2009). Furthermore, in Bower and Falkner's (2015) study, most TCs indicated that they were to some degree not confident about teaching CT, mostly due to issues in pedagogy, technology, and affective issues.

A Growth Mindset. According to Hatsell, Herron, Fang, and Rathod (2010), teacher educators must "address teachers' struggles with certain technologies or mathematical concepts immediately" (p. 57). With the purpose of fostering a positive attitude toward coding and math teaching and in response to the TCs' observed apprehension of the subject matter, the first online week included infographics depicting ideas related to a growth mindset, and TCs reflected on how they could be applied to CT and mathematics teaching.

This experience appeared to be important in the Course, as TCs started to make statements that indicated efforts toward accepting challenges, obstacles, and criticism in their own learning process. Note the following sample comments they made about their own learning process:

> [CT in math education] is challenging and will bring forward great struggle for learners (students and teachers acting as co-learners). Therefore, as one overcomes these struggles they will master [CT in math education] as well as strengthen their growth mindset.

> Thinking about growth mindset challenges me to adopt a different perspective and have a better attitude when I make mistakes or struggle with a new concept such as coding.

> In learning about adopting a growth mindset, I became more optimistic with the course in general and accepting of any mistakes that I may make along the way, knowing that it is a learning process.

The topic/actor of growth mindset appeared consistently throughout the mind-maps, especially in Week 2 (see Table 3), and many TCs (102 out of 143) commented about it in their reflection assignments.

Although most TCs expressed a positive attitude in learning how to code and how to teach mathematics through coding, two TCs continued to express apprehension in Weeks 6 and 8, as shown in the following comment:

> I am putting forth an honest effort to keep a positive and growth mindset but it is becoming increasingly difficult. Part of me really does see the mathematics in coding but the other part of me is weary to teach it.

Comments like this one highlight the importance of affective considerations when designing teacher education for CT.

The concept of the growth mindset is closely related to fostering a conducive environment in the classroom. According to Barr and Stephenson (2011), a growth mindset is beneficial for any learning experience with emphasis on CT, because it includes "acceptance by both teachers and students of failed solution attempts, recognizing that early failure can often put you on the path to a successful outcome" (p. 52). TCs expressed interest in fostering a growth mindset for their students, as in the following example:

> I want to foster a growth mindset in my students, and when I introduce [CT in math education], I will be honest about my experience, and let them know that it was challenging at first, but it is a process, and we will have fun learning together.

Such comments are consistent with the Bower and Falkner (2015) study, where teachers identified their roles in teaching CT not only as instruction providers but also as conducive learning environment creators.

Role of the Online Resources

As mentioned earlier, the online component (the equivalent of 4 out of 9 weeks) consisted of two elements: (a) access to online resources through readings and viewings related to each week's topic, and (b) discussion through collaborative mind-map construction in small groups (four to seven participants per group). Each online week TCs were prompted to complete a list of readings and viewings, which included book chapters, articles, videos and tutorials, to carry out CT activities, and to share their learning through the online mind maps. Resources included videos of keynote addresses by experts (such as Celia Hoyles, Yasmin Kafai and Richard Noss) from a recent Math+Coding Symposium (http://researchideas.ca/coding/proceedings.html), modules from the site What Will You Do in Math Today? (http://www.researchideas.ca/wmt/c6.html, and interviews and articles from the *Math+Code Zine* (http://researchideas.ca/mc/).

Based on TC comments about the online resources, we identified two themes/actors: (a) online resources as tools for their own learning, (b) and online resources as inspiration for their own teaching practice. Table 4 shows how these comments were distributed throughout the mind-maps and assignments.

Table 4
Frequencies of Expressions Related to the Online Resources

Types of Expressions	Week 2	Week 4	Weeks 6 and 8	Reflection Assignments	Total
Usefulness of a resource for their own learning	10	8	11	87	116
Usefulness of the resources as models for TC teaching practice	4	7	24	108	143

Usefulness of a Resource for Their Own Learning. As shown in Table 4, in 116 mind-maps and assignments TCs commented on the usefulness of these materials for their learning, emphasizing the real-life implementations of the activities, and the reflections prompted by educator and mathematician interviews. Following are some representative comments:

> To have real testimony from other teachers of the usefulness of coding was great and also motivating.

> A good point that was brought up by George Gadandis in the interview videos is that we use digital technologies every day, but very few of us know how to code/program; perhaps if we learn more about coding, we can be smarter about our devices and use them more effectively.

> I liked the research ideas website use of introducing the language of coding in a regular problem. For basic problems like what is your favourite food they used when, if, and repeat, which are all used in Scratch as well.

Furthermore, in some cases, the readings would prompt TCs to try and practice the skills they learned in class or to do something new, as is evident in the following comment:

> As I began reading A Coding Story I was trying to figure out multiple ways I could make a square before finishing the article. I was able to create the square doing the typical --move 100 steps, turn 90 degrees (repeat); however, I attempted many other ways and was unable to complete it.

Usefulness of Resources as Models for TC Teaching. In 143 mind-maps and reflection assignments, TCs commented on how the resources would help them as future teachers. Following are some sample comments in which TCs expressed the intention to use some of the lessons portrayed in resources:

> I will definitely remember this website and use it in practicum or my own teaching to help students enhance their learning and understanding.

> [It] is something I can see myself using in the classroom. It is a fun resource and has other opportunities than just math.

As expected, the role of the different resources in TCs' learning was very important. This result is consistent with Bower and Falkner's (2015) research, where preservice teachers highlighted the need for more resources in improving their confidence and ability to teach CT. Also, Barr and Stephenson (2011) suggest that teacher educators "provide teachers with resources to support change, including curricular materials, models and simulations, model activities, and web sites for independent student activities" (p. 53)

Role of the Face-to-Face Experiences

The face-to-face sessions consisted of hands-on learning, using different coding platforms and digital tangibles to develop coding activities in the context of mathematics teaching and learning. The highlights of the face-to-face sessions in TCs' comments were when working with Scratch and Sphero. As shown in Table 1, 95 mind-maps and assignments included comments about in-class activities. Within this sample, two distinctive themes were identified (see Table 5).

Table 5
Frequencies of Expressions Related to the Face-to-Face Experiences

Types of Expressions	Week 2	Week 4	Weeks 6 and 8	Reflection Assignments	Total
Development of positive attitude	8	6	8	53	75
Conceptual understanding	2	2	3	13	20

Development of Positive Attitude. Most TCs who commented about in-class activities expressed positive outcomes and feelings of excitement and fun related to in-class activities (75 out of 95). This assertion is evidenced in the following sample comments:

> When a student's code works, it provides a sense of happiness and accomplishment. I noticed this when working with Scratch and Sphero in class.

> In class we were shown a very complicated coding process that resulted in a very cool picture and pattern to be created. This is a very cool way to look beyond coding as only relating to mathematics, and I can imagine catching the eye and interest of many students, myself included.

> My favourite activity we have done in class was our work with Scratch. [It] is a very cool introduction to coding. It’s user friendly and very easy to use.

> When we had the opportunity to use the Sphero in class, I personally had a lot of fun. It was satisfying to create a code and then watch the Sphero run through the code and complete all the actions I had created.

As seen in these comments, TCs generally described in-class activities as fun, enjoyable, cool, exciting, and satisfying. Some of them also commented on how they would implement what they learned with their own students, foreseeing they would also have a positive experience with the activities. This element was important for shaping their attitudes toward CT and math education. These results echo previous studies that show how in-class implementations of CT activities were attitude boosters toward computing and computer science (Lambert & Guiffre, 2009; Gadanidis, Hughes, Minniti, & White, 2017), even for learners who were initially skeptical about programming activities (Clark et al., 2013).

Conceptual Understanding. As shown in Table 5, five groups in their mind-maps and 13 individual TCs in their reflection assignments described ways that in-class activities and the manipulation of digital tangibles helped develop their understanding of various mathematics concepts. See the following sample comments:

> Similar to what we did in class, by giving students the chance to create different shapes using coding, it can help deepen their understanding of the formulas and concepts regarding shapes, angles and geometry as a whole.

> Something I’m thinking after looking through different Sphero lessons.... Trying to find ways to incorporate them into curriculum makes me approach the knowledge I have in a different way, and it forces me to think about the concept creatively. Creating lessons this way...I think it helps the teacher understand the concept in a deeper way.

The expressions of TCs about in-class activities are consistent with findings of Baytak and Land (2011), who showed how the implementation of computing activities in class improved skills such as designing and problem solving. However, the conceptual understanding and problem solving skills had considerably lower frequencies of appearance as compared to the development of positive attitudes (see Table 5).

Most TC comments focused on how the activities were enjoyable and fun rather than on the conceptual understanding derived from these activities. This result is consistent with Bower and Falkner’s (2015) findings among teachers who showed a “focus on tool usage for engagement, rather than a deep understanding of computational thinking processes and concepts" (p. 40).

What TCs Thought With

The Theoretical Framework section described our effort to identify the actors that TCs came to think with in their learning process as they engaged with the various ideas and teaching practices presented in the Course. We analyzed TCs' online collaborative mind-maps and individual reflection assignments to identify themes in four categories (which match the four research questions): (a) what TCs learned about CT in math education; (b) what attitudes they developed; (c) what role was played by online resources; and (d) face-to-face classroom activities. The themes we identified in each of these categories, which reflect the actors that TCs came to think with, are summarized in Figure 3.

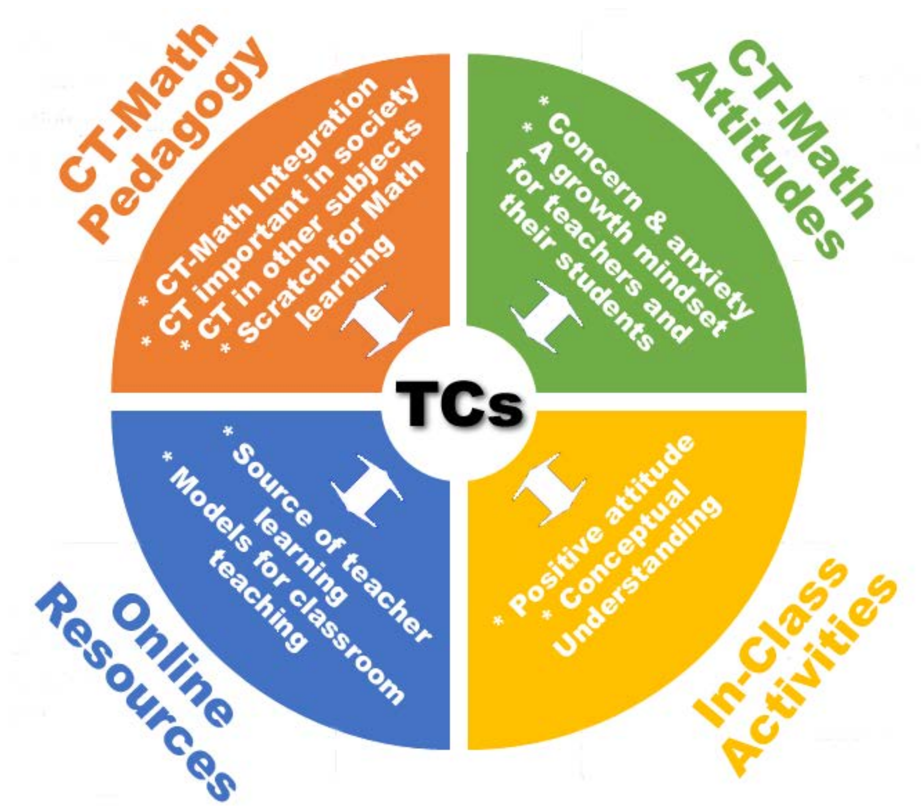

Figure 3. *The actors that TCs thought with.*

What TCs Learned About CT in Math Education. TCs appear to have made much progress in this short, 18-hour course. Most TCs came to the Course with little or no knowledge of CT and even less knowledge of CT-math integration. Our analysis of what they shared and discussed in their mind-maps and in their reflection assignment indicates that the Course helped them develop new ideas to think with.

We identified three CT-integration actors evident in TC online discussions: CT-math integration; Scratch in math teaching; and CT in other subjects. We also identified the importance of CT in society as an actor, which supports and provides a rationale for the importance of the other three actors. As we will elaborate later in this section, we cannot make claims about the extent to which these actors may influence TC thinking beyond the Course. Also, in retrospect, it should be expected that a course on CT in Mathematics Education would prompt TCs to think with such CT-actors.

What Attitudes TCs Developed. With respect to the second question, we identified two attitude-actors: (a) an apprehension about CT-math integration; and (b) a growth mindset towards CT-math integration, for themselves and for their students. Not surprisingly, the apprehension-actor was prominent in the early stages of the course. Elementary school TCs typically have arts and humanities backgrounds and do not have a strong background in mathematics, even less in CT-related fields. However, as the course progressed, as TCs experienced hands-on CT-math activities, watched online videos of students and teachers engaging with similar ideas at the classroom level, and read professional and scholarly literature related to CT and mathematics education, the growth-mindset-actor became more prominent in the thinking evident in their online discussions.

The Role Played by Online Resources. We have been building online resources for mathematics teaching for over 10 years. Over the last 4 years, we have also been developing CT-math online resources. These resources offer teaching ideas with lesson plans and classroom artifacts. Sometimes, they are also accompanied by videos of classroom action, with teacher interviews and even with interviews of mathematicians working through the same activities as elementary school students and discussing them from their perspective, such as the Probability in Grade 1 videos available at the following site: http://researchideas.ca/wmt/c6b2.html.

The two resource-actors we identified, namely the teacher-learning-actor and the pedagogical-modelling-actor, are consistent with our study of similar resources we use in mathematics education courses for elementary school TCs (Gadanidis & Cendros Araujo, in press).

The Role of Face-to-Face Classroom Activities. In our mathematics education courses, we make it a priority to model and to offer TCs hands-on experiences with activities that they may use in their own classrooms. Given the limited course time we have available to teach about curriculum, pedagogy, mathematics, and so forth, we focus our attention on providing experiences that offer conceptual surprise and insight and to help TCs see mathematics, and CT, in a fresh light. The two face-to-face-actors we identified, namely the positive-attitude-actor and the conceptual-understanding-actor, paralleled our findings for hands-on activities we use in mathematics education courses for elementary school TCs (Gadanidis & Cendros Araujo, in press).

Superficial vs Pervasive Actors. Kaplan (1991) distinguished between superficial and deep beliefs and cautioned that not all statements made by teachers equally translate into classroom practice. Similarly, we can distinguish between superficial and pervasive actors. That is, the influence of the actors identified in this study may be limited to online course discussions, and their influence may not translate to subsequent classroom practice. Alternatively, the actors may provide TCs the language of change but not its translation into practice. Arguably, when TCs move to new settings, such as from course discussions to practicum classrooms, new actors may come into play which may have greater or competing influence.

CT-actors already exist in classroom settings today, however, due to the various calls for coding from industry, government, academia and nonprofit sectors. At the same time, there is a gap at the classroom level between these calls and their practical implementation in an already crowded curriculum. In this context, the actors identified in Figure 3 may find opportunities to exert influence.

Our anecdotal evidence indicates that at least some TCs thought with the actors in Figure 3 in their teaching. For example, immediately following the completion of the Course, TCs began their second practicum placement in schools. Several TCs contacted the instructor via email to request ideas for teaching specific math strands using CT. They tried several Course activities in their practicum class and experienced success. Most of these activities were related to geometry concepts, where TCs had their students code 2-D shapes using Scratch and Sphero and pseudocode (unplugged) activities, such as following written commands.

TCs who used CT in their teaching reported that their associate teachers encouraged them to continue integrating CT in math class. As well, TCs requested to borrow the digital making and robotics materials used in their Course. About 10 TCs borrowed materials to integrate CT ideas in math classrooms. The Course instructor noticed phrases that indicated excitement in their emails and informal face-to-face discussions. Some TCs also helped to organize math nights at their practicum schools and incorporated coding activities they had developed as a result of their experiences in the Course. Subsequently, some classroom teachers who hosted TCs have contacted the instructor to request in-service sessions.

Concluding Remarks

CT has entered the educational landscape, and it is crucial that faculties of education programs understand how it will impact teaching and learning and how they might address the added knowledge that teachers need to develop. In Canada, the provinces of Nova Scotia and British Columbia have announced that they will be introducing CT as a curriculum objective across grades K-12. Other provinces are likely to follow suit in the near future. Internationally, the curriculum shift toward CT is also evident, with the 2014 introduction of a coding curriculum for all K-12 students in England, to give one example.

CT in mathematics education is not new. It was an integral part of the work of Papert (1980) with Logo. The curriculum focus on CT at the moment appears to view CT as its own curriculum objective, rather than integrated to support and enhance learning of existing subject areas, as was the case with Logo and mathematics. However, "there is a natural (and historical) connection between computational thinking and mathematics—in terms of logical structure and the ability to model mathematical relationships" (Gadanidis et al., 2017, p. 77). CT integration also affords new approaches to mathematics problem-solving, and broadens the range of mathematics with which students can engage (Buteau, Gadanidis, Lovric & Muller, 2017).

This paper offered a case study of a first implementation of a CT Course for K-6 TCs with a focus on mathematics education. Looking ahead, we plan to continue this research by conducting a longitudinal study of the Course. We also plan to expand the research to investigate TC practice in their practicum settings: (a) whether/how TCs think with Course ideas (such as those in Figure 3) in their practicum teaching, and (b) whether/how these ideas may transfer to the teaching practice of other teachers in their practicum settings.

Computational thinking, in the form of coding, digital making, and in unplugged settings, appears to have captured the interest and imagination of TCs and teachers we work with. More research is needed to understand this phenomenon and its implications for mathematics teacher education and for teaching and learning mathematics.

References

Barr, V., & Stephenson, C. (2011). Bringing computational thinking to K-12: What is involved and what is the role of the computer science education community? *ACM Inroads, 2*(1), 48-54.

Baytak, A., & Land, S.M. (2011). An investigation of the artifacts and process of constructing computer games about environmental science in a fifth grade classroom. *Educational Technology Research and Development, 59*, 765-782

Berg, B.L. (2004). *Qualitative research methods for the social sciences*. New York, NY: Pearson Publishers.

Bers, M.U., Flannery, L., Kazakoff, E.R., & Sullivan, A. (2014). Computational thinking and tinkering: Exploration of an early childhood robotics curriculum. *Computers & Education, 72*, 145-157. doi:10.1016/j.compedu.2013.

Borba, M.C., & Villarreal, M.E. (2005) *Humans-with-media and reorganization of mathematical thinking: Information and communication technologies, modeling, experimentation and visualization*. Nova Iorque: Springer Science+Business Media, Inc.

Bower, M., & Falkner, K. (2015). Computational thinking, the notional machine, pre-service teachers, and research opportunities. *Proceedings of the 17th Australasian Computing Education Conference (ACE 2015)*, 37-46. Retrieved from http://crpit.com/confpapers/CRPITV160Bower.pdf

Buteau, C., Gadanidis, G., Lovric, M., & Mueller, E. (2017). Computational thinking and mathematics curriculum. In S. Osterle, D. Allan, & J. Holm (Eds.), *Proceedings of the 2016 annual meeting of the Canadian Mathematics Education Study Group Conference* (pp. 119-136). Queen's University, Kingston, ON, Canada.

Clark, J., Rogers, M.P., Spradling, C., & Pais, J. (2013) What, no canoes? Lessons learned while hosting a Scratch summer camp. *Journal of Computing Sciences in Colleges, 28*, 204-210.

Creswell, J.W., & Miller, D.L. (2000). Determining validity in qualitative inquiry. *Theory into Practice, 39*(3), 124-130. doi:10.1207/s15430421tip3903_2

Curzon, P., McOwan, P.W., Cutts, Q.I., & Bell, T. (2009). Enthusing and inspiring with reusable kinaesthetic activities. *ACM SIGCSE Bulletin*, 94-98.

Denzin, N.K. (1989). *The research act: A theoretical introduction to sociological methods* (3rd ed.). Englewood Cliffs, NJ: Prentice Hall.

Gadanidis, G. (2017). Five affordances of computational thinking to support elementary mathematics education. *Journal of Computers in Mathematics and Science Teaching 36*(2), 143-151.

Gadanidis, G., & Cendros Araujo, R. (in press). Teacher candidates' online math journals: A search for pedagogical surprise. *International Journal of Mathematics Education Research*.

Gadanidis, G., Hughes, J., Minniti, L., & White, B. (2017). Computational thinking, grade 1 students and the Binomial Theorem. *Digital Experience in Mathematics Education, 3*(2), 77-96.

Hartsell, T., Herron, S.S., Fang, H., & Rathod, A. (2010). Improving teachers' self-confidence in learning technology skills and math education through professional development. *International Journal of Information and Communication Technology Education, 6*(2), 47-61.

Kaplan, R.G. (1991). Teacher beliefs and practices: A square peg in a square hole. *Proceedings of the 13th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, 119-125), Blacksburg, VA.

Knochel, A.D., & Patton, R.M. (2015). If art education then critical digital making: Computational thinking and creative code. *Studies in Art Education, 57*(1), 21-38.

Koh, J.H., & Divaharan, S. (2011). Developing pre-service teachers' technology integration expertise through the TPACK-developing instructional model. *Journal of Educational Computing Research, 44*(1), 35-58.

Lambert, L., & Guiffre, H. (2009). Computer science outreach in an elementary school. *Journal of Computing Sciences in Colleges, 24*(3), 118–124.

Latour, B. (2005). *Reassembling the social: An introduction to Actor-Network-Theory*. Oxford, UK: Oxford University Press.

Levy, P. (1993). *Tecnologias da Inteligência: O futuro do pensamento na era da informática*. [Technologies of Intelligence: the future of thinking in the informatics era]. Rio de Janeiro, Brazil: Editora 34.

Levy, P. (1998). *Becoming virtual: Reality in the digital age*. New York, NY: Plenum Press.

Maeng, J.L., Mulvey, B.K., Smetana, L.K., & Bell, R.L. (2013). Preservice teachers' TPACK: Using technology to support inquiry instruction. *Journal of Science Education and Technology, 22*(6), 838-857.

Meijer, P.C., Verloop, N., & Beijaard, D. (2002). Multi-method triangulation in a qualitative study on teachers' practical knowledge: An attempt to increase internal validity. *Quality and Quantity, 36*(2), 145-167. doi:10.1023/A:1014984232147

Miles, M.B., Huberman, A.M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Thousand Oaks, CA: SAGE Publications, Inc.

Mishra, P., & Yadav, A. (2013). Of art and algorithm: rethinking technology and creativity in the 21st century. *TechTrends, 57*(3), 10-14. doi:10.1007/s11528-013-0668-7

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York, NY: Basic Books, Inc.

Stake, R.E. (2000a). Case studies. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed.; pp. 435-454). Thousand Oaks, CA: Sage Publications.

Stake, R.E. (2000b). The case study method in social inquiry. In R. Gomm, M. Hammersley, & P. Foster (Eds.), *Case study method: Key issues, key texts.* London, UK: Sage Publications.

Thumlert, J., de Castell, S., & Jenson, J. (2014). Short cuts and extended techniques: Rethinking relations between technology and educational theory. *Educational Philosophy and Theory*, *47*(8), 786-803.

Voogt, J., Fisser, P., Good, J., Mishra, P., & Yadav, A. (2015). Computational thinking in compulsory education: Towards an agenda for research and practice. *Education and Information Technologies, 20*(4), 715-728.

Vygotsky, L.S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Wing, J. (2006). Computational thinking. *Communications of the ACM, 49*(3), 33-36.

Yadav, A., Mayfield, C., Zhou, N., Hambrusch, S., & Korb, J.T. (2014.). Computational thinking in elementary and secondary teacher education. *ACM Transactions on Computing Education, 14*(1), 1-16.

Yin, R. (1994). *Case study research: Design and methods* (2nd ed.). Beverly Hills, CA: Sage Publishing.

Contemporary Issues in Technology and Teacher Education is an online journal. All text, tables, and figures in the print version of this article are exact representations of the original. However, the original article may also include video and audio files, which can be accessed online at http://www.citejournal.org