

Research On Graphing Calculators at the Secondary Level: Implications for Mathematics Teacher Education

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Abstract

This article focuses on three key factors that a survey of literature indicated impact the teaching and learning of mathematics with graphing calculators: access to graphing calculators, the place of graphing calculators in the mathematics curriculum, and the connection between graphing calculators and pedagogical practice. Access to graphing calculators is associated with student achievement gains and a wide array of problem-solving approaches. The research suggests students' achievement is positively affected when they use curricula designed with graphing calculators as a primary tool. Studies of teachers' use and privileging of graphing calculators illustrate the impact professionals have on students' mathematical knowledge and calculator expertise. Implications of these research findings for preservice and in-service teacher education are summarized.

Graphing calculators were first introduced in 1985 and within a few years mathematics educators began to study the role and impact of this tool on the teaching and learning enterprise. The field has amassed a significant body of research on students' performance and learning with graphing calculators and a small, now growing, body of research on teachers' use and knowledge of graphing calculators. An analysis of research studies published in peer-reviewed journals over the past 2 decades suggested a framework for summarizing the findings and implications of this research. Three themes emerged that cut across the existing literature: access to graphing calculators, the place of graphing calculators in mathematics curricula, and the connection between graphing calculators and pedagogical practice.

This article addresses what this literature suggests about teaching and learning mathematics with graphing calculators, as well as the implications of these research findings for preservice and in-service teacher education.

Access

Traditionally, mathematics has been taught as a collection of rules and procedures that make computations more efficient. Thus, it comes as little surprise that in a context where the focus of mathematical activity is computation access to tools that can perform many of these computations has historically been restricted. The studies discussed in this section illustrate how teachers' beliefs and knowledge influence access to graphing calculators and how, in turn, this access influences students' mathematical performance.

In a 1994 survey of close to 100 middle and secondary mathematics teachers, just over 70% of those surveyed indicated that they had calculators available for classroom use but had not used them in their own classes (Fleener, 1995a, 1995b). This study, and studies by others (Doerr & Zangor, 1999, 2000; Slavit, 1996), have consistently shown that merely providing teachers with access to graphing calculators does not ensure that students then have access. Access to graphing calculators, even for students who own them, is mediated by the teacher.

The question of whether students will be given access to graphing calculators is often connected with teachers' beliefs about the roles graphing calculators should play in the learning process (Doerr & Zangor, 1999, 2000; Fleener, 1995a, 1995b; Leatham, 2002). Limited access is most often associated with a belief that graphing calculators should be used only after students have mastered a particular mathematical procedure by hand and then primarily as a means of checking ones' work. Frequent access is usually associated with a belief that graphing calculators should be used to facilitate the understanding of a mathematical concept. Leatham (2002) found that access to graphing calculators was a critical concern for preservice secondary mathematics teachers and was one of the primary dimensions of the participants' core beliefs about technology use in the classroom. These beliefs about access ranged from desiring extremely limited access to desiring that students have access at all times. These beliefs were similarly associated, respectively, with procedural and conceptual objectives.

In addition to demonstrating that teachers' beliefs about graphing calculators influence student access to graphing calculators, researchers have shown how access to graphing calculators influences student performance. Harskamp, Suhre, and van Streun (1998; 2000; van Streun, Harskamp, & Suhre, 2000) conducted a study designed to compare the performances of students with differing levels of access to graphing calculators (TI-81). They compared the performances of calculus students in 12 classes who were randomly assigned to one of three groups: those with no access to graphing calculators, those who had access during one unit of instruction (approximately 6 weeks), and those with access for one year. Although all students studied function and calculus concepts from a common textbook, the graphing calculator groups received additional instructions on how to use the calculators to perform several tasks. Graphing calculators were used to check algebraic solutions, to find solutions graphically, and to graph functions. More advanced operations were not explored. None of the 12 teachers assigned to these classes had prior experience teaching with the graphing calculator.

Students were given pre- and posttests designed to assess their problem solving strategies. All students were allowed to use scientific calculators on the pretest. The

control group was allowed to use scientific calculators on the posttest, while the experimental groups were allowed to use graphing calculators. The test was designed so that having a graphing calculator would not be an advantage. Four problem solving strategies, drawn from the work of Kieran (1992), were used to code the students' work: heuristic, graphical, algorithmic, and no solution or unknown. Heuristic strategies are those involving trial and error. Graphical strategies depend on the creation of graphs. Algorithmic strategies are based on algebraic procedures, such as computing the derivative. Although scores on the pre- and posttests were not statistically significantly different, the authors did see differences in the approaches used by students.

Results showed that students with the longest access to calculators used a wider range of problem-solving approaches and "tended to attempt more problems and obtain higher test scores than the students who had not" (Harskamp et al., 2000, p. 37). In addition, students referred to as "below average" by the researchers made more frequent use of graphical strategies and "achieved a significantly higher mean posttest score ($p < 0.05$) than students in the control group" (pp. 47-48). Students using the calculators for one unit also used more graphical strategies than they had on the pretest (van Streun et al., 2000). These students tended to replace heuristic and algorithmic strategies with graphical approaches.

This collection of reports suggests that even limited access to calculators may have a positive effect on students' approaches to mathematical problems. This interpretation is supported by the findings of other studies (Adams, 1997; Hong, Toham, & Kiernan, 2000) conducted with students whose access to calculators was limited. Perhaps the shortest period of calculator access that produced student performance gains was reported in Hong, Toham, and Kiernan (2000). In the study, the performances of two groups of New Zealand calculus students were compared on a series of tests. The experimental group was taught to use calculators (TI-92) with computer algebra systems (CAS) in four 1-hour lessons. Students in the control groups were taught integration using a traditional approach. On a posttest consisting of traditional university entrance exam questions and on which all students were allowed to use a non-CAS graphing calculator, students in the experimental group outperformed those in the control group. Thus, access to calculators, even limited access, appeared to result in improved student performance.

Based on the findings from these two studies, one might be tempted to simply supply calculators for students and assume that, provided teachers allow the access, scores will increase. Additional findings of Hong et al. (2000), however, suggest that short-term access may bring problems that are not apparent in these comparisons. When Hong et al. (2000) compared students' performance on a calculator neutral test, designed to measure conceptual growth rather than computation skill, students in the control group outperformed those in the experimental group. In addition, Hong et al. (2000) investigated the impact of the calculators on the performance of students they referred to as low achieving and found that the calculators enabled students to complete computational problems that they could not do on the pretest. This use resulted in large gains for these students but, as the authors suggested, no new understanding of calculus. Based on these study results, we learn that some students with short-term access may experience hollow performance gains.

Additional problems associated with limited access to calculators were illuminated in the report of a study conducted with eighth-grade general education students (Merriweather & Tharp, 1999). The students used calculators (TI-82) in class for 2 weeks to complete traditional algebra problems. The authors noted, "The majority of the students felt they were not comfortable with the calculator and did not use it" (p. 19). For these students the calculator caused confusion; consequently, they tended to use problem-solving strategies

that were more familiar to them, such as working backwards. Although the performance of some students may improve when they have short-term access to calculators and are taught using traditional instructional materials, research indicates gains do not appear to be conceptual. In addition, students with limited access may become confused and overwhelmed as they attempt to integrate their knowledge of mathematics with their developing understanding of a new tool.

In contrast to these negative findings associated with short-term access, Graham and Thomas (2000) conducted a study in New Zealand in which algebra students (13-14 years old) in treatment and control groups were taught for 3 weeks. The treatment groups were taught using TI-82 or Casio FX7700GH calculators and a 3-week module, *Tapping into Algebra*, designed to provide opportunities for students to develop their understanding of variables in algebraic expressions (symbolic literals). The curriculum was designed “to use the graphic calculator’s lettered stores as a model of a variable” (p. 269). The store function allows a letter to be assigned a constant value. For example if $A = 3$, then students would be asked to make conjectures about the result of $A + 3$. Later students were asked to assign values to two letters and to predict results of expressions such as AB , $2A + 2B$, and $4(A + 5B)$. Also included in the unit were investigations of “squares and square roots, sequences, formulas, random numbers and function tables of values” (p. 270). The control groups were taught the same topics with a focus on whole-class instruction and skill development. Students were given one posttest designed to “measure understanding of the use of letters as [a] specific unknown, generalized number and variable” (p. 272) and another designed to measure procedural skills. Students in the treatment groups outperformed those in the control group on the test of use of symbolic literals and performed as well as those in the control groups on the test of procedural skill.

Two factors in the Graham and Thomas study (2000) provide plausible reasons why this short-term access had a more positive impact on students’ performance than that reported in the previously discussed studies. First, the students may have had access to either scientific or graphing calculators prior to the treatment. Student comments reveal that at least some participants had experience with scientific calculators. For example, one student noted he or she liked “the way the screen showed all the numbers coming up. I found it much easier than all the other calculators which don’t even show the number” (p. 276). Experience with a scientific calculator may have had an impact on the success of the module. Second, and more importantly, the calculator was not simply added on to the standard curriculum. Instead, the authors of *Tapping into Algebra* developed the content of the module with the tool in mind. Thus, the results suggest that another critical factor in teachers’ use of and student performance and learning with the graphing calculator is the role of the tool in the curriculum.

Curriculum

Researchers in mathematics education assert that understanding a mathematical concept, such as function, includes the ability to use and make connections between multiple representations (Confrey & Smith, 1991; Hiebert & Carpenter, 1992). Although developing more than one representation for a mathematical object can take a substantial amount of time, effective use of graphing calculators allows quick and easy development of and translation between representations. Curriculum developers were quick to realize the power of these tools and to design investigations that made use of the device as a lever to learn mathematics (Schwarz & Hershkowitz, 1999). This section discusses the results of studies that investigated student learning with curricula designed to take advantage of access to graphing calculators (Schwarz & Hershkowitz, 1999).

The curricula of the Core-Plus Mathematics Project (CPMP) and the second edition of the University of Chicago School Mathematics Project (UCSMP) were developed to take advantage of graphing calculators. The developers of CPMP worked from the premise that graphing calculators provide access to mathematics that has heretofore required proficiency in skills such as symbol manipulation (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000). Graphing calculators afford students the opportunity to explore problems using graphical, numeric, and symbolic strategies and to make links between these strategies (Core-Plus Mathematics Project, 2004). The UCSMP curriculum was developed to meet the mathematical and technological needs of individuals and our society as suggested by mathematicians and mathematics educators (Usiskin, 1986). While the first edition materials required access to scientific calculators, the second edition materials were “influenced by advances in technology, particularly the availability of graphing calculators” (Thompson & Senk, 2001, p. 60).

Studies designed to identify the effect of these curricula on student achievement (Huntley et al., 2000; Thompson & Senk, 2001) found that students taught using the curricula outperformed those taught using traditional approaches on application problems (see example in Figure 1). However the findings of the two studies differ with regard to student performance on procedural tasks such as the one illustrated in Figure 2. On tests comprised of procedural items, Huntley et al. (2000) found Algebra I students who had used traditional curricula outperformed students who had used CPMP; Thompson & Senk (2001) found no statistically significant difference between the performances of their groups on a similar test.

Although it is unclear from these studies which factors produced the performance gains on application

problems, interpretation of results from two other studies (Boers & Jones, 1994; Ruthven, 1990) provide some plausible explanations. By contrast, these two studies involved curricula not designed to take advantage of graphing calculators. The results of these studies suggest that the intent of curricular materials significantly influences the way graphing calculators are used in the classroom.

Boers and Jones (1994) examined student work on a traditional calculus test. The students and teacher used a traditional text; however, students were also given instruction on the use of the calculator (TI-81) in “graphing, solving equations and inequalities, the numerical evaluation of derivatives and integrals, picturing and evaluating limits, and numerically checking analytically derived mathematical solutions” (p. 492). Analysis of student solutions revealed that students “made minimal use of the

When a baseball is thrown straight up from a height of 5 feet with an initial velocity of 50 ft/sec, its height h in feet after t seconds is given by the equation $h = -16t^2 + 50t + 5$. Assuming that no one catches the ball, after how many seconds will the ball hit the ground?

Figure 1. Application problem (Thompson & Senk, 2001, p. 84)

The solutions to $5x^2 - 11x - 3 = 0$ are...

Figure 2. Procedural task (Thompson & Senk, 2001, p. 82)

calculator outside of questions requiring a specific graphical response” (p. 494) and had difficulty integrating graphical information they generated into their solutions.

For example, when students were given a rational function and asked to find the values for which the function was not defined, to find the limit of the function as x approached a given value, and to graph the function, only 5 of the 37 students integrated algebraic and graphical information to produce their solution. Generally, students “treated questions as either essentially ‘algebraic’ or ‘graphical’ (or as having distinct algebraic and graphical parts), depending on what the questions asked for” (Boers & Jones, 1994, p. 514). One plausible interpretation of this finding is that students may have learned how to apply calculator techniques as suggested in the supplemental lessons and how to apply algebraic techniques as suggested in the text, but not how to integrate the information derived from these two sources to solve problems. This disconnect between graphing calculators and the curriculum impeded students’ ability to integrate the various techniques they had learned.

Ruthven (1990) also studied the use of graphing calculators with curriculum materials not designed to take advantage of those tools. He investigated the effect of calculator access on upper secondary students’ ability to translate graphs into algebraic form and to interpret graphics of contextual situations. In this curricular context, Ruthven found that students who used graphing calculators (fx-7000) outperformed students without access to the graphing calculators on symbolization tasks (e.g., given the graph of a quadratic function, find an algebraic representation). There was no statistically significant difference between scores of the two groups of students on tasks that asked them to make interpretations from graphical representations of contextual situations (e.g., given a time versus speed graph for a cyclist, determine when the speed will be greatest). Ruthven suggested that this lack of distinction was due to the fact that the curriculum materials were of little use in preparing students to draw interpretations from graphs.

The results from these two studies suggest that students for whom the graphing calculator is simply added on to traditional texts develop algebraic and graphical approaches to mathematics problems. Unlike students using problem solving investigations developed to be used with graphing calculators, students using calculators as an add-on were unable to integrate mathematical information drawn from different representations. Such students were no better than their peers with access to traditional texts and scientific calculators at interpreting mathematics problems in context. Inconsistent gains illustrated by Boers and Jones (1994) and Ruthven (1990) are characteristic of instruction in which students are asked to make sense of an inconsistent curriculum.

The results of these studies do not imply, however, that only large-scale curriculum projects can produce healthy, connected student use of graphing calculators. Indeed in each of the four studies discussed, the authors suggested that the method of instruction was a critical factor in determining student learning. This phenomenon, often referred to as the difference between the intended curriculum and the “curriculum in practice” (Romberg, 1992, p. 750), leads to a discussion of the influence of pedagogy on classroom teaching and learning with graphing calculators.

Pedagogy

In addition to providing materials designed to take advantage of access to graphing calculators, UCSMP and CPMP curricula assumed instruction would consist of investigations of novel problems by groups of students who explored and discussed problem situations and worked cooperatively toward solutions. These solutions were seen

as the basis for the mathematics the students discussed as a whole class and ultimately learned. This approach stands in contrast to the pedagogical techniques used in the classrooms involved in the research of Boers and Jones (1994) and Ruthven (1990). Although teachers in these latter studies welcomed and used graphing calculators in their classrooms, they attempted to teach using traditional lecture-style approaches. Students from this pair of studies appear to have learned very different mathematics. This section discusses research on the impact of pedagogy on student learning with graphing calculators.

To investigate how pedagogy might impact student learning, Kendal and Stacey (1999) studied three teachers' approaches to teaching calculus with the graphing calculator (TI-92) and their Year 11 students' approaches to solving problems with the technology. The three teachers and several researchers "designed a twenty-five lesson introductory calculus program, with a focus on differentiation, that aimed to use CAS to enhance conceptual understanding, connections between representations, and appropriate use of the technology" (p. 234). Students and teachers were experienced users of the TI-83, but were novice users of the TI-92.

Researchers found that, although the teachers had agreed to teach the lessons using the same approach, three different emphases were apparent: Teacher

From the corners of a rectangular piece of cardboard, 32 cm by 12 cm, square sides of side, x cm, are cut out and the edges turned up to form a box. Find the value of x if the volume of the box is a maximum.

Figure 3. Problem to introduce maximum/minimum (Kendal & Stacey, 1999, p. 241)

A made frequent use of the calculator and algebraic approaches; Teacher B "preferred the traditional algebraic approach using graphs when essential" (p. 243); and Teacher C emphasized the use of algebraic and graphical methods and the connections between them. The difference in the three teachers' approaches was most obvious in the researchers' descriptions of approaches they used to introduce the same maximum/minimum problem (see Figure 3). Teacher A drew a diagram, used a volume formula, and demonstrated how to solve the problem using a CAS procedure. Teacher B guided the students to the development of a function for the volume of the box and through the by-hand solution method. Teacher C guided the students through the development of a function and used student suggestions as the basis for graphical and CAS solutions. She also demonstrated algebraic and graphical procedures using the projection unit linked to a graphing calculator.

The average test scores for the three classes were similar, but an item-by-item analysis revealed "differences between classes with respect to the calculator use and success on items" (p. 236). Although students in Class A attempted more items, in general, and used the calculator more often, they had a lower overall success rate on the items they attempted than did students in the other two classes. Students in Class B "preferred to use by-hand algebra" (p. 239) and applied these techniques with success; they failed to use CAS, however, "when its use would have been advantageous" (p. 244). Students in Class C were discriminating about their use of the calculator and "compensated for their poorer algebraic skills by substituting a graphical for an algebraic procedure" (p. 245). These students did not use CAS to differentiate polynomial functions as students in Class A did. In an analysis of student conceptual errors, errors that cannot be avoided by using CAS, the error rate for Class C was 4.9 errors per student as compared to 7.3 and 5.7 for

students in classes A and B, respectively. Thus, although the differential emphases in the classes did not produce significant differences in overall student performance, a closer look at students' approaches to various problems revealed that the performance of students in each class was closely aligned with the pedagogy of their teachers.

Similar results were found by Porzio (1999) in a study of students' self-selecting into three different sections of calculus: traditional text and lecture approach, text developed to be used with a graphing utility and a graphing calculator, and a Mathematica section taught without a text. The author found that "students 'behave' as they are taught" (p. 25), solving problems using algebraic, graphical, or multiple representations depending on the representations their teachers presented most often. Interpretation of these results suggests that when access to graphing calculators and curriculum developed to use these tools are controlled, the critical factor in student learning is pedagogy.

Implications for Mathematics Teacher Education

Thus far this paper has focused on three common themes in research on the use of graphing calculators with secondary mathematics. As teachers largely determine students' access to graphing calculators, the curriculum that is actually enacted, and the pedagogical landscape of the classrooms, the findings of this research have serious implications for the role of teacher education in ensuring success in the learning and teaching of mathematics with graphing calculators. Organized around these three common themes, the following sections illustrate ways in which teacher education programs can inform teachers of and be informed by this research.

Access to Graphing Calculators

As has been discussed previously, mere access to graphing calculators does not ensure an impact on teaching and learning. In order for teachers to be prepared to take advantage of access to technology, teacher education programs must help teachers feel knowledgeable and comfortable with technology. This can be done in a number of ways. First, teachers need early and frequent access to graphing calculators. They need experience both learning and teaching with graphing calculators in constructive learning environments.

Second, the focus of such efforts should go beyond the functionality of the tool to incorporate the potential and the constraints of the tool. In order to make thoughtful decisions about when and how to use the tool, teachers need to be given opportunities to discuss the benefits as well as the possible pitfalls of using graphing calculators in their classrooms. It is through such discussions that teachers can explore and develop their own beliefs about student access to and use of graphing calculators and the implications of such access.

Third, teacher education should expose teachers to research on the effects of access to graphing calculators. This opportunity will enable them to develop beliefs that are both informed by current research and connected to their own mathematical understandings. When experiences learning and teaching with graphing calculators are coupled with exposure to research in these areas, teachers revisit the instructional approaches they use or plan to use to teach mathematics. These reflections may, in turn, motivate teachers to provide the kind of access to graphing calculators advocated by current movements in mathematics education reform (e.g. National Council of Teachers of Mathematics, 2000).

Curriculum

Teacher education programs need to address issues related to the place of graphing calculators in the curriculum. Teachers often feel restricted by their curriculum—in particular, by high-stakes testing. If teachers perceive that these tests value by-hand procedures, then they feel obligated to prepare their students to perform on these tests and, for teachers, this often means limiting the use of graphing calculators. Teacher education programs should familiarize teachers with curricula, such as UCSMP and CPMP, which have been developed to take advantage of graphing calculators as a tool for developing mathematical understanding.

Exposure to research on the use of graphing calculators with various curricula can influence teachers' decisions with respect to curricular demands. Although results from the studies examined herein are mixed, there is evidence that students using graphing calculators in problem solving develop understanding of mathematics that allows them to answer traditional test items successfully (Thompson & Senk, 2001). Examples such as this suggest it is possible to use all the tools at hand, including graphing calculators, to help students gain deep understandings of important mathematics (National Council of Teachers of Mathematics, 2000) while still successfully fulfilling curricular demands.

Pedagogy

Research reports contain numerous suggestions for how teacher education programs can effectively influence teachers' pedagogical decisions regarding the use of graphing calculators. We discuss here three main categories of these suggestions: reflection, experience, and learning to manage inherent challenges.

Teacher education programs should give preservice and in-service teachers opportunities to reflect on their personal philosophies and beliefs about graphing calculators (Fleener, 1995a, 1995b), combined with reflection on their beliefs about mathematics, its teaching and learning (Simmt, 1997). Professional development can facilitate reflection by employing multiple strategies, such as participating in classroom discussions, writing in journals (Tharp, Fitzsimmons, & Ayers, 1997), and both sharing and listening to personal experiences learning and teaching with graphing calculators (Simonson & Dick, 1997).

Opportunities to reflect should go hand in hand with opportunities to experience teaching and learning mathematics with graphing calculators. One way for teachers to have such experiences is for them to observe videotaped classrooms in which teachers are teaching and students are learning with graphing calculators (Tharp et al., 1997). Reading research reports such as those synthesized for this paper can also provide teachers with opportunities to observe, through the written word, examples of how teachers have effectively used graphing calculators in their classrooms (Doerr & Zangor, 1999, 2000; Goos, Galbraith, Renshaw, & Geiger, 2000; Slavitt, 1996). Other studies (Drijvers & Doorman, 1996; LaGrange, 1999a, 1999b; Zarzycki, 2000) provide illustrations of lessons teachers developed that provide opportunities for their students to investigate mathematics that is inaccessible without the use of graphing calculators. These studies provide vivid examples of how technology “influences the mathematics that is taught and enhances students' learning” (National Council of Teachers of Mathematics, 2000, p. 24). In addition, exploring the use of technology in the classroom can provide a context for exploring other pedagogical decisions (see Laborde, 1999). Observation of effective graphing calculator use should be coupled with substantial mathematics explorations in which teachers experience the benefits of learning with graphing calculators (Fleener, 1995a; Tharp et al., 1997), in particular, the multiple representations graphing calculators

afford. Teachers should then be given opportunities to experiment with these strategies in their teaching.

Research has documented and articulated a number of challenges inherent in pedagogical approaches that involve the use of graphing calculators. Teacher education programs should give teachers opportunities to explore implications of these challenges and ways in which they can be addressed. Lessons involving explorations with graphing calculators are often less structured, requiring teachers to share control of the classroom with their students (Goos et al., 2000; Tharp et al., 1997). Teachers should become familiar with the disadvantages as well as the advantages of using graphing calculators in their classrooms (Hong et al., 2000). Discussions about the limitations of graphing calculators can be valuable learning experiences for teachers as well as for their students (Mitchelmore & Cavanagh, 2000). Finally, Schmidt (1998) offered concrete examples of how teachers can deal with external factors, such as standardized tests, and pressures from parents and state and district mandates.

This paper has considered some common themes in the research on teaching and learning with graphing calculators and implications of such research for teacher education. There is a need, however, for more far-reaching research on the use of graphing calculators, built on efforts to integrate research on the teaching and learning of mathematics (e.g., Fennema, Carpenter, & Lamon, 1991). Research on the use of graphing calculators in the classroom needs to move to a new phase of complexity, where teacher education, teacher use, student use, and student learning are all taken into account. This may be challenging, but it is necessary if research is to produce the kind of knowledge base about teaching and learning with graphing calculators that, in the face of constant technological advancement, will continue to inform teacher education and practice effectively.

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