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Online Mentoring with the Math Forum: A Capstone Experience for Preservice K-8 Teachers in a Mathematics Content Problem-Solving Class

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Abstract

This article describes how the Problem of the Week Environment at the Math Forum online mathematics resource allows K-8 preservice teachers who are enrolled in mathematics content problem solving-classes to experience the process of reading, evaluating, and replying to young problem solvers' work with thoughtful comments and effective hints. This online project includes the training of college-student mentors, the assignment of problems, and the approval of replies. This article focuses on the twofold purpose of the mentoring project: first, to give preservice teachers a special type of field experience by guiding K-8 students to write better solutions via questions and helpful suggestions; and second, to allow preservice teachers the opportunity to reflect upon the variety and richness of approaches generated by a rich mathematical problem.

Problem Solving Courses and the Problem of the Week Environment

The National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000) focuses on problem solving as the basis of learning and conceptualizing new mathematical knowledge and as a fundamental motivating factor for studying mathematics. The NCTM (2000) *Principles and Standards for School Mathematics* clearly stated the following:

> Instructional programs from pre-kindergarten through grade 12 should enable all students to—
>
> - Build new mathematical knowledge through problem solving;
> - Solve problems that arise in mathematics and in other contexts;
> - Apply and adapt a variety of appropriate strategies to solve problems;
> - Monitor and reflect on the process of mathematical problem solving. (p. 52)

This emphasis, in turn, means that teacher education programs must take very seriously the task of preparing teachers who have a solid mathematics knowledge base in problem solving and a deep understanding of how new mathematical knowledge is generated, enhanced and illustrated through the problem solving process (Erickson, 1999; Martinez & Martinez, 2000; Posamentier, 2004; Trafton, Midgett, & Joyner, 2001).

Problem-solving courses for K-8 preservice teachers at both Hope College and Western Oregon University (WOU) are offered as upper level courses, following the foundational mathematics content course sequences at both schools. These upper level courses aim to improve the problem solving abilities of the college students through posing, discussing, and solving challenging problems. They focus on helping the college students internalize Polya's (1971) framework by providing numerous opportunities for exploring common story problem paradigms and their theoretical extensions. College students are expected to be actively involved in class exploration activities that are done in cooperative groups and to master a problem-solving assessment rubric (Muth, 1997; Robinson & Bartlett, 1995; Santos-Trigo, 1998). There is a continued emphasis on discourse, on writing clear and complete explanations, and on creating and adapting worthwhile mathematical story problems (Green, 2002; Klein, 2001).

In the first part of their respective course work, preservice teachers at both Hope College and WOU focus on improving and expanding their problem-solving skills. At the end of the upper level courses on problem solving, they then get to apply their knowledge of mathematics and the problem-solving process through their participation in online mentoring with the Math Forum.

The Math Forum website (http://mathforum.org/) is a valuable resource for engaging students at all levels in problem solving. A variety of archived problems is available online; new problems become available weekly throughout the academic year as part of the Problem of the Week (PoW) Environment (http://mathforum.org/pow).

The PoW Environment poses nonroutine challenging problems to students from around the world. These problems are organized by level and topic: Math Fundamentals PoW (late elementary school), Pre-algebra PoW (middle school), Geometry, and Algebra (typically aimed at algebra I level high school students). The Archive and Active Problem Library also contain problems in trigonometry, precalculus, calculus, and discrete mathematics. Young problem solvers (submitters) have a 2-week window after a problem is first posted to email a solution that includes a full explanation of each step used to

obtain the final answer to the posed questions. If submitters leave a reflective comment after checking their numeric answer, their chances of receiving a reply from a mentor are high. The mentor's role is to read carefully through each assigned submission, assess the work according to a published problem-solving rubric, and write a short reply with appropriate acknowledgements and helpful hints.

Typically, the Math Forum operates this mentorship system with a group of experienced educators called the Cavalry Group. Members of this elite team reply to submitters directly. When preservice teachers participate in the Online Mentoring project, their replies to problem solvers are filtered through an "approver" (who is typically either the course instructor or an experienced Cavalry member) before these replies are sent out.

As Margaret Ford (1994) pointed out, "The need for reflection as teachers integrate mathematics content knowledge, pedagogy, and individual beliefs is a key to the process of becoming an effective teacher of mathematics" (p. 322). As mentors in the PoW environment, the preservice teachers have a unique opportunity to interact online with young problem solvers. They are also able to engage in a reflective, mathematically meaningful activity that can serve as a unifying content-pedagogy experience in a mathematics problem solving class. On the one hand, preservice teachers get to think about the mathematics of the posed problem and to read a variety of different solutions. On the other hand, they get to "teach" young submitters how to be better problem solvers, through writing a series of probing questions and interesting hints in a friendly, nonthreatening way.

As mentors, preservice teachers easily understand how this experience will prepare them for the future, when they will each be the responsible teacher. They learn firsthand about multiple solution methods and the young minds that produced them. They are surprised by young submitters on both ends of the learning spectrum. They encounter those whose writing skills make any conclusive interpretation difficult and others whose mastery of the vocabulary of mathematics and attention to detail are astounding. The international nature of the experience also allows the mentors to view our own educational system through a different lens.

This project was established when mathematics instructors at both Hope College and WOU proposed that their preservice teachers act as mentors in the Math Forum environment. The experience included the following steps:

1. The college students wrote or selected a "rich" story problem and then suggested it for consideration in the PoW environment.
2. The college students completed an online training session to become mentors.
3. The college students mentored actual young students engaged in solving the problem.
4. The college students helped to compose final commentary on the problem.
5. The college students reflected upon and evaluated their overall experience.

Preparations for Online Mentoring

The training of college student mentors consisted of working through a series of lessons posted on a WebCT board at Drexel University. The first lesson introduced college students to a demonstration problem of the week. In order to experience the process through which young problem solvers go, all college students proceeded to work on this problem and to write a solution online. The preservice teachers then read online several of the solutions submitted by their peers. A class discussion generated an Expected

Solution to the demonstration problem. This lesson could be completed online, or the discussion portion of it might be held face to face.

In the next lesson, the college students were invited to mentor each other. As novice mentors, their first attempts at mentoring invariably generated a lively discussion about how to be an effective mentor, and they also led into a study of the Mentoring Guidelines established by the Math Forum.

In subsequent lessons, college students were introduced to the problem solving scoring rubric. This rubric has two main sections: problem solving and communication. The problem solving section has interpretation, strategy, and accuracy as its components. Similarly, the communication section includes clarity, completeness, and reflection. Problem solvers are given a rating of novice, apprentice, practitioner, or expert in each of the six categories.

As the college students became familiar with the scoring rubric, they engaged in practice scoring for the demonstration problem of the week. Comparing their results with those from the Math Forum team was an important part of the learning process. Following that activity, they began to read and discuss mentor replies to submitted solutions for that same problem. They also read the comments that approvers sent back to the mentors. Careful reading was important as the college students played the role of approvers and as they discussed and critiqued the mentored replies.

Finally, the preservice teachers were ready to study the expected solution of the new problem they would mentor. The mentoring guidelines were reviewed again, and after a class discussion and wrap-up of the training, the first replies were assigned for mentoring.

Online Mentoring

Each college student was assigned between 5 and 20 replies, each in its own "thread." The exact number depended on the number of submissions to that particular PoW, and the number of college students serving as mentors. College students were responsible for replying to their assigned threads in a timely manner. In those cases where the submitters decided to revise their work, the same college students were responsible for evaluating the revised solutions, as well.

Before a reply written by a preservice teacher was sent back to a submitter, it was evaluated by an approver. As mentioned earlier, typically the instructor of the class and perhaps an experienced student helper handled such approvals. Approvers ensured that scoring was assigned correctly and consistently, that the tone of the reply was appropriate, and that all mathematical information was correct. Approvers did one of the following:

1. Requested a revision by the preservice teacher.
2. Edited minor errors and then sent the reply.
3. Simply sent the reply as written.

The structure and composition of a well-written reply were important topics for in-class discussion. Class discussion addressed such topics as a common format, questioning strategies, the nature of praise or constructive criticism (including the tone of the reply), and finally, the "nit-picking details" such as grammar, spelling, and punctuation.

Deciding early what constitutes a well-written reply is a time-saver for professors and/or approvers as they seek to evaluate preservice teachers' mentoring work.

The scoring rubric addresses well the nature of an “Expert” solution. College student mentors learned that praise needed to be earned in order to be authentic. Phrases like "great job" should mean that someone has indeed done so. This eliminated the danger of the young problem solvers not reading further into the mentor's reply. Preservice teachers could be enthusiastic in other ways that are genuine. "Thanks for working on this problem" was a way to encourage submitters who may not have achieved a correct solution or clear explanation. Criticism needed to be somewhat specific in order to help the problem solver, with the expectation that the problem solver was capable of fixing the situation, as the following quote illustrates: "You started this problem in a way that makes sense to me, but I don't understand why you divided by 2 in the third step. Perhaps you could explain that in a revision." Sometimes the problem solver's work would be correct and would require a more precise explanation. At other times there might be mathematical mistakes, but the rubric scoring process helped to clarify the difference for the submitters. Appendixes A and B include examples of mentoring.

Post-mentoring

After all the solution submissions were mentored, the preservice teachers engaged in writing a summary commentary for the problem. This was another outstanding opportunity for preservice teachers to reflect on the problem and to appreciate the variety of approaches undertaken by submitters. Commentary for the Building Bouquets problem (http://mathforum.org/funpow/solutions/solution.ehtml?puzzle=213) is found in Appendix C.

Conclusion

As might be expected the preservice teachers at both Hope College and Western Oregon University enjoyed this project tremendously. The following is a short list of typical college students' comments:

- "The PoW mentoring was worthwhile because it gave me firsthand experience with correcting math problems. It made me think about responding from a teacher's perspective."
- "I would recommend that lots of other colleges adopt this plan of evaluating problems."
- "Overall I think I learned a lot about problem solving through my participation in the Math Forum. I knew there were different ways of solving problems before, but actually reading how students think about a problem and reading their exact solution puts everything into a new perspective."
- "I think the Math Forum Mentoring project was an eye-opening experience of what I could possibly expect from my future students."
- "I think the Math Forum Mentoring project was a great experience that was a lot of fun and gave me a great opportunity to apply the problem solving skills that I have been learning all term."
- "In fact, I have learned that the answer is not the most important aspect to a solution. How an answer is arrived at and how it is verified will tell much more about how a student actually understands a problem. I am now much more confident to move closer to being a teacher after this experience."

In conclusion, we believe that preservice teachers benefited greatly from this mentoring experience. This ongoing project can serve as a natural capstone experience in a problem-solving class because it has allowed college students to contemplate the depth and richness of the problem solving process through the task of reading and evaluating the actual work of young problem solvers. It has also familiarized preservice teachers with the spectrum of thinking, writing, and reasoning skills that their future students will bring into the classroom. Furthermore, it has given college students a small, though unforgettable, taste of how difficult yet rewarding the task of reaching out and teaching mathematics to young minds can be.

The nature of the online environment was an important aspect of this experience. When interacting face-to-face in a classroom field experience, preservice teachers must make immediate responses to student actions or questions. Such quick responses may fail to address the complexity of the mathematics involved. In busy classrooms with active youngsters, the time for really *listening to* and evaluating the problem solver's approach to a problem may be a luxury that does not exist. Also, the college students may need reflection time in order to understand a novel approach. Sometimes, they may need to study the related mathematical concepts in order to truly answer a question. The online environment slowed the interaction down to provide the preservice teachers with important time to "think before they speak." It allowed them to study the mathematics and to make careful responses that would help the young learners to improve their mathematical problem solving and communication.

The technology also allowed for genuine interactions in a way that is adaptable to many different situations. Because of the round-the-clock availability, college students could insert the mentoring into their schedule where it fit. The mentoring experience was an ideal field experience when barriers such as transportation or illness might prevent the college student from participating in a more traditional practicum.

Another benefit for the mentoring experience has been the exposure to the wealth of mathematics available at the Math Forum. The authors' regular interaction with the staff of this established website has convinced them that problem selection and assessment are being done carefully and intentionally, with important consideration given to the research about its teaching and learning. The problems used for PoW are carefully refined prior to being posted, and they exhibit the depth of content that the college students should experience prior to their teaching careers.

Throughout the training and mentoring process, the preservice teachers learned valuable mathematical lessons. They strengthened skills in student interaction and fueled their enthusiasm for their classroom careers.

References

Erickson, D. (1999). A problem-based approach to mathematics instruction. *The Mathematics Teacher, 92*(6), 516-521.

Ford, M. I. (1994). Teachers' beliefs about mathematical problem solving in the elementary school. *School Science & Mathematics, 94*(6), 314-323.

Green, D. A. (2002). Last one standing: Creative, cooperative problem solving. *Teaching Children Mathematics, 9*(3), 134-140.

Klein, M. (2001). Constructivist practice, preservice teacher education and change: The limitations of appealing to hearts and minds. *Teachers & Teaching, 7* (3), 257-271.

Martinez, J. R., & Martinez, N. C. (2000). Teaching math with stories. *Teaching PreK-8, 30*(4), 50-54.

Muth, D. K. (1997). Using cooperative learning to improve reading and writing in mathematical problem solving. *Reading & Writing Quarterly, 13*(1), 72-85.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Polya, G.(1971). *How to solve it.* Princeton University Press.

Posamentier, A. S. (2004). Marvelous math! *Educational Leadership, 61*(5), 44-49.

Robinson, G., & Bartlett, K.T. (1995). Assessing mathematical learning. *Teaching Children Mathematics, 2*(1), 24-27.

Santos-Trigo, M. (1998). Instructional qualities of a successful mathematical problem-solving class. *International Journal of Mathematical Education in Science & Technology, 29*(5), 631-647.

Trafton, P., Midgett, C., & Joyner, J. (2001). Learning through problems: a powerful approach to teaching mathematics. *Teaching Children Mathematics, 7* (9), 532-538.

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Appendix A
Example of Mentoring 1

This is an example of the mentoring work of a preservice teacher who naturally falls into the role of a mentor. Her reply is approved without any corrections.

1.1 The problem is the Movie Seating Problem (http://mathforum.org/prealgpow/solutions/solution.ehtml?puzzle=213).

> A family of six goes to see a movie at the downtown movie theater. The family includes Mom, Dad, and the four kids – Randy, Shelley, Noreen, and Alan. They all sit in a row. Use the clues below to figure out the order in which they sit.
>
> a. Dad wants to sit as far away as possible from Randy because last time Randy spilled soda pop on him.
> b. Noreen never sits on the outside because she does not like sitting next to strangers.
> c. Alan and Shelley need to be separated by at least two people (and one of them has to be a parent), or else they will spend the entire show talking.
> d. Unfortunately, Mom does not get to sit right next to Dad.
> e. During the movie Alan whispers a secret about Randy into Noreen's ear. There are three seats between Alan and Randy.
> f. Randy is sitting at one of the ends.

Here is a solution submitted by a seventh-grader:

> **Answer:**
>
> The family at the movies are in this order from left to right; Dad, Alan, Noreen, Mom, shelley, and Then Randy.
>
> **Explanation:**
>
> The very 1st step I did was make 6 lines and represent them as the family's 6 seats. It looked like this... ___ ___ ___ ___ ___ ___. When I finished making the lines I recopied all the clues under the 6 lines so I wouldn't have to keep looking up at the computer screen. Then I rearranged all the family's seating orders to fit the clues given. Therfore the seating order (from left to right) is Dad, Noreen, Alan, Mom, Shelley, and Randy. This is my conclusion for the "Movie Seating" pre-algebra problem of the week.

1.2 Mentoring reply:

> Dear _____,
>
> Thank you for your submission to the Movie Seating problem. I really liked how you said that you used the 6 lines to represent the seating of the family. This is a great idea.

You told me what strategy you used to solve the problem, but I would really like to see each step that you went through and how you used each of the clues to place each family member. For example, how did clue (a) help you to place some of the family members?

One other thing that you might want to do is go back through your solution when you are finished and check to make sure that the clues fit your solution. By doing this you can make sure that your solution is correct; this is a good habit to get into.

You are off to a good start. I recommend that you resubmit your solution and try for a Practitioner score in all of the categories. If you have any questions or get stuck, please do not hesitate to ask for help.

Good luck!

Summary:

Problem Solving Interpretation:	Practitioner
Strategy:	Practitioner
Accuracy:	Practitioner
Communication Completeness:	Apprentice
Clarity:	Apprentice
Reflection:	Novice

(for an explanation of scores see: http://mathforum.org/pow/scoring.html)

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Appendix B
Example of Mentoring 2

This is an example of the work of a preservice teacher who needs to revise her original mentoring work. Both her original and revised replies are included, together with the letter from the approver that indicates the necessary changes to the original reply.

2.1 Here is a solution by a 10-year old to the Movie Seating Problem:

> **Answer:**
>
> Dad is sitting on the far left. Next to him on the right is Alan. Next to Alan on the right is Noreen. Next to Noreen on the right is Mom. Next to Mom on the right is shelly and next to Shelly on the right is Randy.
>
> **Explanation:**
>
> First I read the first clue and found out that Randy needs to be in one of the ends and Dad needs to be on the other end. Then I read the next clue in which I found out that Noreen needs to be in one of the 4 middle seats. Then I read the next clue and found out that Alen and Shelly need to be separated by at least two people and one of them has to be Mom. So I put Alen and Shelly each on one end next to the two far seats (one next to Dad and one next to Randy). I read the next clue and saw that Mom cannot be right next Dad. Then I read the next clue and found out that Alen Wispers a secret into Noreen's ear which means that Alen must be sitting next to Noreen. I also found out from the same clue that there are three seats between Alen and Randy. Then I read the last clue and found out that Randy is sitting on one of the ends.

2.2 Original mentoring reply from a college student mentor:

> Dear _____,
>
> Thank you for taking the time to submit to the Problem of the Week. You have solved the problem correctly.
>
> If you are interested in improving there are a couple of things you might want to consider. First, have someone (a classmate for instance) proof read your explanation before you send it. Second, you might want to address whether or not your solution works according to the clues given. If it does, go ahead and mention how it is you checked the solution. Also, while I follow your explanation, you might consider expanding on how it is you know for certain that Mom and Noreen sat where they did. These are just a couple things you could do if you wanted to improve.
>
> If you have any questions or are interested in improving on the work you have already completed please don't hesitate to do so. In fact I am anxious and do hope to hear back from you. Best of luck!

Summary:

Problem Solving Interpretation:	Apprentice
Strategy:	Novice
Accuracy:	Practitioner
Communication Completeness:	Apprentice
Clarity:	Apprentice
Reflection:	Novice

(for an explanation of scores see: http://mathforum.org/pow/scoring.html)

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2.3 Approver's comments:

Hi!

I thought that you have made very good suggestions; I just have a couple for you.

I would suggest that you ask questions instead of just stating what they should do. For instance instead of saying, "have someone proofread your explanation..." You could say, "Do you think you could have a classmate proofread your explanation so that the steps you took are clear to them?" By asking a question it gets the students more involved.

Also, I would at least give them apprentice on strategy, they are not a novice. They have used the clues to solve the problem. Also, Interpretation should be a practitioner score in my opinion. They interpreted it right and tried to solve it.

Thanks, keep up the good work.

2.4 Revised mentoring reply by the college student mentor:

Dear _____,

Thank you for taking the time to submit to the Problem of the Week. You have solved the problem correctly.

If you are interested in improving there are a couple of things you might want to consider. First, do you think you could have a classmate proof read your explanation so that the steps you took are clear to them? Second, you might want to address whether or not your solution works according to the clues given. If it does, go ahead and mention how it is you checked the solution. Also, while I follow your explanation, you might consider expanding on how it is you know for certain that Mom and Noreen sat where they did. These are just a couple things you could do if you wanted to improve.

If you have any questions or are interested in improving on the work you have already completed please don't hesitate to do so. In fact I am anxious and do hope to hear back from you. Best of luck!

Summary:

Problem Solving Interpretation:	Practitioner
Strategy:	Apprentice
Accuracy:	Practitioner
Communication Completeness:	Apprentice
Clarity:	Apprentice
Reflection:	Novice

(for an explanation of scores see: http://mathforum.org/pow/scoring.html)

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Appendix C
Commentary for the Building Bouquets Problem

It was very interesting for our group to mentor this problem. We enjoyed reading solutions from so many different places, and we feel the experience has given us valuable insight to use in our teaching careers.

In most of the solutions we read, submitters made lists or a table to help them compare the multiples of 4, 7, and 8. The solution from _____ shows how listing the multiples plus the leftovers works. They did a great job including their table, and we were impressed that they found a different approach for the Extra when their lists became too long.

The submission from _____ is also very good. _____ shows the multiplication with the addition of the remainder in her lists, and we especially like the advice she gives Jana: if you plan ahead, you just might save yourself some work down the road.

_____took a slightly different approach though still used lists organized in a table. Notice how she found common multiples of two of the numbers first (4 and 8), then added the remainder of three, and finally checked to see which result worked with the third number (7). _____ also did a nice job reasoning through the Extra.

A few submitters used guess and check to solve the puzzle. This is a useful strategy, but it's often not explained well. When you use guess and check, be sure to include your incorrect guesses and say how they helped you reassess to make a better next guess... and eventually find the correct answer. In other words, you need to show more than a check that your final guess worked (and if you were lucky with your first guess you need to say that, but also include some numbers that don't work to check your results - or maybe find others!).

To find success with this problem it was necessary to recognize multiples, organize your work well, and check your results to be sure they fit all the requirements of the situation. Congratulations to those of you who did this and explained your thinking well. To those of you who had trouble solving the puzzle or writing a complete explanation, we encourage you to keep working - we're sure you'll continue to improve as long as you keep practicing and revising.

Thanks for sending submissions and revisions!